Name:____

Do Now: **Find the derivative of the following equation with respect to t, (d/dt) if v, r and h are variables**

$$V = \pi r^2 h$$

Related Rates

Many things change with time. Our goal is to find the rate at which some quantity is changing by relating the quantity to other quantities whose rates of change are known

Mr. C's steps to solving related rate problems:

1) Draw a picture if applicable

2) Identify what quantity you are looking for

3) Identify the given information; the rates $\frac{d(variable)}{dt}$ can often be identified by the problem stating changing, increasing or decreasing. If the rate is decreasing be sure to make the quantity negative.

4) Take the derivative of the equation that relates the variables with respect to "t" (remember to make use of the derivative rules) before you plug the given information in

5) Substitute the given rates and values into the derivative equation

6) Use the original equation before you took the derivative to solve for any other variables necessary to find the desired quantity you were originally looking for.

7) Solve for the desired quantity

General

The power P (watts) of an electric circuit is related to the circuit's resistance R (ohms) and current I (amperes) by the equation $P = I^2 R$.
Given: R=5 ohms, P=45 watts, I is *decreasing* at $\frac{1}{3}\, amps/sec$, R is *increasing* at 2 $ohms/sec$
find $\frac{dP}{dt}$.

1. Given Information:

2. Looking for:

3. Equation to be used

4. Derivative of the equation:

5. What is "I" equal to:

6. Solve:

Circle

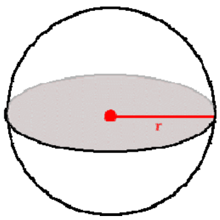
When a circular shield of bronze is heated over a fire its radius increases at the rate of $\frac{1}{5}$ cm/sec.
At what rate is the shields area increasing when the radius is 50 cm?

1. Given Information:

2. Looking for:

3. Equation to be used:

<u>Sphere</u>

	The **<u>volume</u>** of a sphere is given by the equation: $$V = \frac{4}{3}\pi r^3$$	The **<u>surface area</u>** of a sphere is given by the equation: $$S = 4\pi r^2$$

Question: A spherical snowball with an outer layer of ice melts so that the *volume* of the snowball decreases at a rate of 2 cm^3/min.

(a) How *fast is the radius changing* when <u>diameter</u> of the snowball is 10 cm?

1. Given Information:

4.

2. Looking for:

3. Equation:

(b) How fast is **surface area** of the snowball *decreasing* at this time?

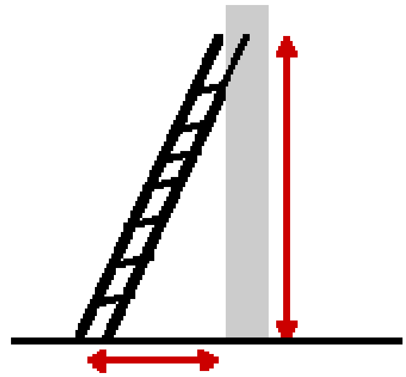
1. Given Information

4.

2. Looking for:

3. Equation:

<u>Right Triangles</u>
A *ladder 20 feet long* leans against a vertical house. If the bottom of the ladder slides away from the house *horizontally at a rate of 4 ft/sec*, how *fast is the ladder sliding down* the house when the *top of the ladder is 8 feet from the ground*.



(a) How are these variables related?

(b) What is the given information?

(c) What are you looking for?

(d) What is the derivative of this equation?

(e) What is $\frac{dz}{dt}$?

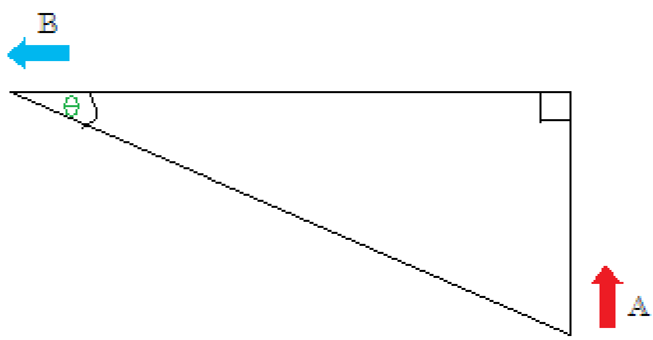
(f) What is x at this time?

<u>Advice from Mr. C:</u>

1) Since the length of z does not change, $\frac{dz}{dt} =$
2) Remember, not to plug the variables until you find the derivative
3) Then go back to the original $x^2 + y^2 = z^2$, to find the missing side of the right triangle

Right Triangles:

Two students A and B are walking on straight roads that meets at right angles. Student A approaches that intersection at *2 m/sec* and student B moves away from the intersection at *1 m/sec* as shown in the figure. At what rate is the angle θ changing when *A is 10 m* from the intersection and *B is 20 m* from the intersection? Express your answer in radians per second to the nearest degree.



1. Given Information:

2. Looking for:

3. Equation that relates the variables:

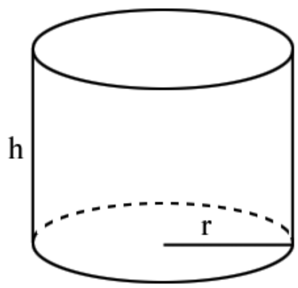
4. The derivative of the equation:

Advice from Mr. C:

1. Remember the chain rule when taking the derivative of the trig function to get $\frac{d\theta}{dt}$

2. Always work in radians until the end of the problem, at which point you can convert to degrees

Cylinder

Right cylinder	The **volume** of a right circular cylinder is $V = \pi r^2 h$	
	If the cylinder does not changes dimensions and water is poured in what happens to	
	(a) $\frac{dh}{dt}$ of the water	(b) $\frac{dr}{dt}$ of the water

1) A right cylindrical tank is filled with water. The tank stands upright has *radius 20 cm*. How *fast does the height of water* in the tank drop when the water is being *drained at 25 cm^3/sec*?

1. Given Information:

2. Looking for:

3. Equation that relates the variables:

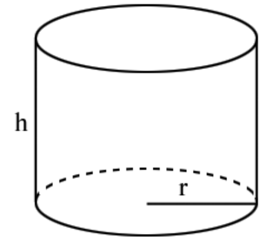
4. The derivative of the equation:

Advice from Mr. C:

1. When taking the derivative of the volume equation use the product rule and think of the equation as is $V = (\pi r^2)(h)$

2. Remember, if the cylinder does not change dimensions, the radius of the substance inside will not change, hence $\frac{dr}{dt} =$

2) The *radius* of a right circular cylinder *increases at the rate of 0.1 cm/min*, and the *height decreases at the rate of 0.2 cm/min*. What is the *rate of change of the volume* of the cylinder, in cm^3/min, when the radius is 2 cm and the volume is 12π cm^3/min?



Right cylinder

1. Given Information:

2. Looking for:

3. Equation:

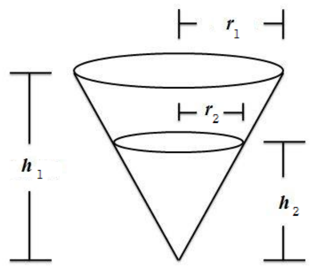
4. The derivative of the equation:

5. How do you find h?

<u>Advice from Mr. C:</u>

In this problem because both the radius and height are change we will have a $\frac{dr}{dt}$ and $\frac{dh}{dt}$

<u>Cone</u>

	The relationship between the radius and height of the cone and the radius and height to the water: $$\frac{r_1}{h_1} = \frac{r_2}{h_2}$$	The volume of a cone: $$V = \frac{1}{3}\pi r^2 h$$ (where r is its radius and h is its height.)

1) A water tank in the shape of a right circular cone has a *height of 10 feet*. The top rim of the tank is a circle with *a radius of 4 feet*. If water is being pumped into the tank at the *rate of 2 cubic feet per minute*, what is the *rate of change of the water depth*, in feet per minute, when the *depth is 5 feet*?

1. Given Information:

2. Looking for:

3. Equation:

4. Use similar triangles to eliminate one variable, since we are looking for , we solve for _______, so when we plug it into the volume equation the _______ goes away :

5. Take the derivative of the new equation

2) Corn is poured through a chute at the rate of 10ft^3/min, and falls in a conical pile whose bottom radius is always half the height.

(a) How fast will the radius of the base change when the pile is 8 ft high?

1. Given Information:

4. Use similar triangles to eliminate one variable, since we are looking for , we solve for _______, so when we plug it into the volume equation the _______ goes away :

2. Looking for:

5. Take the derivative of the new equation

3. Equation:

(b) How fast is the circumference of the base increasing at this time?

<u>Advice from Mr. C:</u>

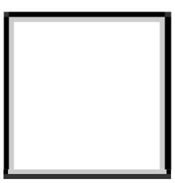
1) Before taking the derivative of the volume equation, you usually have to eliminate one variable r or h. This can be done by setting up the ratio $\frac{r_1}{h_1} = \frac{r_2}{h_2}$

2) If you are looking for $\frac{dr}{dt}$ solve for h in the ratio and if you are looking for $\frac{dh}{dt}$ solve for r.

General

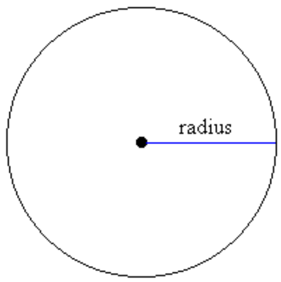
1) Boyle's law for confined gases states that if the temperature is constant, $pv = c$, where p is pressure, v is volume and c is a constant. At a certain instant the volume is 75 in^3, the pressure is 30 lb/in^2, and the pressure is decreasing at a rate of 2lb/in^2 every minute. At what rate is the volume changing at this instant?

2) Let *A be the area of a square* whose sides have length x. Given $\frac{dx}{dt} = 2\frac{ft}{\min}$. How fast is the area of the square increasing at the time when x = 3 ft



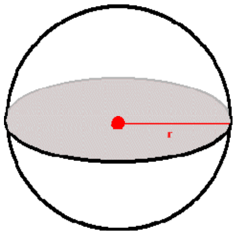
Circle

1) The *radius* of a circle is *increasing* at a constant rate of 2 centimeters per minute. Find the *rate of change of area* when r = 6 centimeters



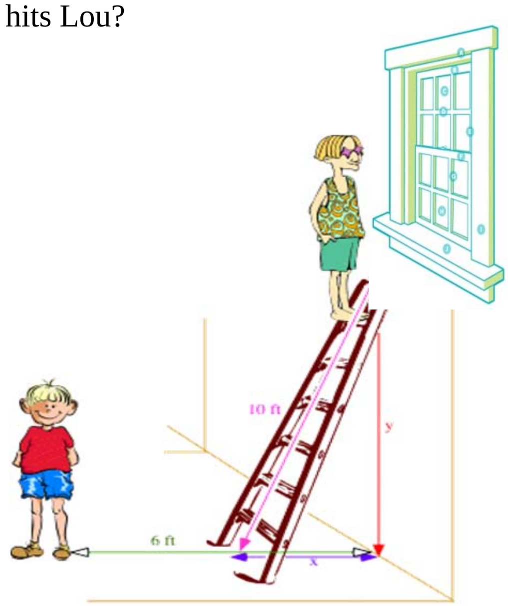
Sphere

1) Gas is being pumped into a spherical balloon at a rate of 5 ft^3/min. Find the rate at which the radius is changing when the diameter is 18 inches.



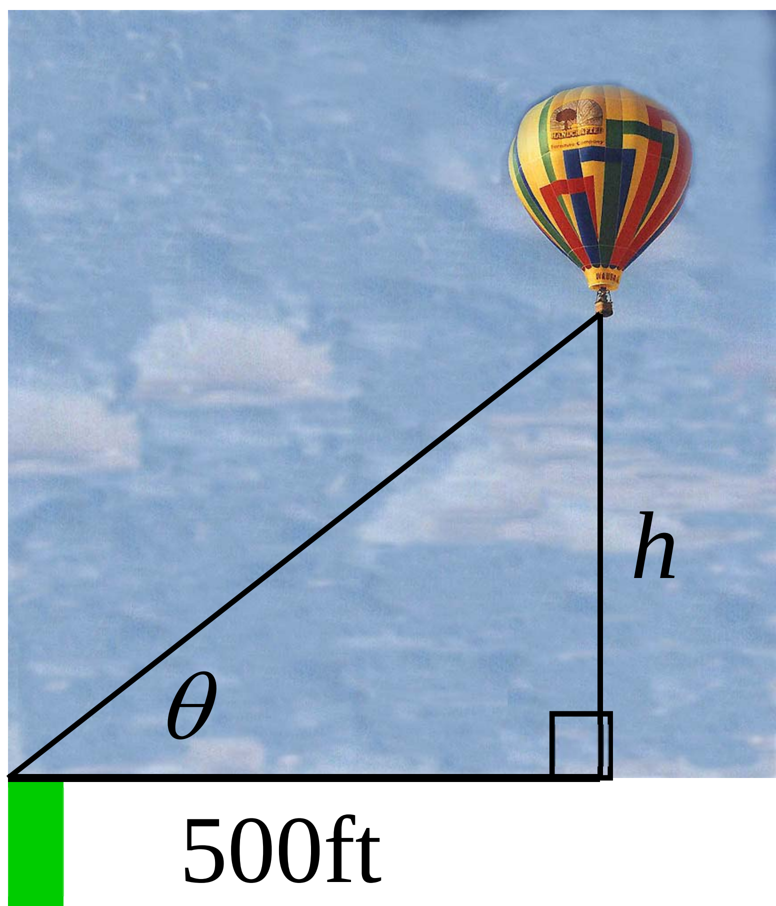
Triangles

Joey is perched precariously the top of a 10-foot ladder leaning against the back wall of an apartment building (spying on an enemy of his) when it starts to slide down the wall at a rate of 4 ft per minute. Joey's accomplice, Lou, is standing on the ground 6 ft away from the wall. How fast is the base of the ladder moving when it hits Lou?



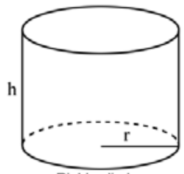
Triangles

$Given: \ \theta = \frac{\pi}{4}, \frac{d\theta}{dt} = 0.14 \ \frac{rad}{min}$ How fast is the balloon rising?



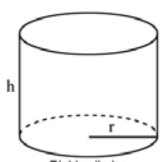
Cylinder

1) A cylindrical tank standing upright has radius 10 cm. How fast does the water level in the tank rise when the water is being poured in at 32 cm^3/sec?



Right cylinder

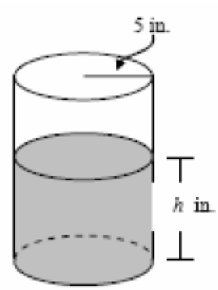
2) The radius of a right circular cylinder decreases at the rate of 0.2 cm/min, and the volume increases at the rate of 6π cm^3/min. What is the rate of change of the height of the cylinder, in cm/min, when the radius is 3 cm and the height is 5 cm?



Right cylinder

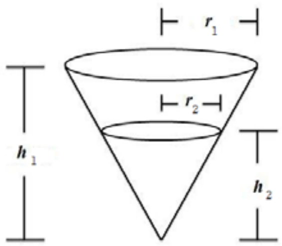
AP Question:

Example: A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure to the right. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.



Cone

1) Gravel is being dumped from a conveyor belt at a rate of 10 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 7 ft high? Give your answer correct to three decimal places.



2) A conical water tank with a diameter of 20 ft at the top and is 24 feet high. If water flows into the tank at a rate of 20ft^3/min, how fast is the depth of the water increasing when the water is 16ft deep?